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PRESENTATION

Minimum Hop Multicasting in Broadcast Wireless Networks with Omni-directional Antennas

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Outline

- **Problem Statement:** Minimum-hop multicasting in wireless networks.
- **Issues:**
  - Individual nodes are equipped with limited capacity batteries and therefore have a restricted communication radius.
- **Results:**
  - Mixed Integer Linear Programming model of the problem.
  - Sub-optimal sequential shortest path heuristic algorithm with “node unwrapping” – amenable to distributed implementation.
  - Simulation results indicate that reasonably good solutions can be obtained using the proposed heuristic algorithm.

Introduction

- Establishing a broadcast/multicast tree in such networks often requires co-operation of intermediate nodes which serve to relay information to the intended destination node(s).
- Minimizing the number of hops in the routing tree is motivated by the need to conserve bandwidth, minimize end-to-end delays – especially for delay-critical data packets – and reduce packet error probabilities.
- In certain military applications, employing a low-power multicast tree with minimum number of transmissions can serve to further reduce the possibility of detection/interception.
- Individual transmissions in multicast trees in these networks are generally low-powered.

Introduction

- A suitable topology control algorithm can be used to ensure a power efficient topology.
- For example, topologies can be constructed to minimize the maximize transmitter power needed to maintain connectivity [Ramanathan, *Infocom 2000*] or the total transmitter power.
- Our focus here is to provide solutions for minimum hop multicasting in power efficient wireless network topologies.
- Previous work includes a Hopfield neural network based approach and a couple of heuristics [Pomalaza-Raez et. al., *TCC 1996*].

Network Model

- Fixed N-node wireless network with a specified source node and a broadcast/multicast application.
- Any node can be used as a relay node to reach other nodes in the network.
- All nodes have omni-directional antennas.
- All nodes have limited capacity batteries which limits the maximum transmitter power and hence the degree of connectivity of a node (number of nodes which can be reached by a transmitting node using a direct transmission).

Network Model

- The power matrix, $\mathbf{P}$, is an $N \times N$ symmetric matrix, the $(i,j)^{th}$ element of which represents the power required for node i to transmit to node j:

$$\mathbf{P}_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{\alpha/2} = d_{ij}^{\alpha} \tag{1}$$

  where

  – $\{(x_i, y_i) : 1 \leq i \leq N\}$ are node coordinates.

  – α ($2 \leq \alpha \leq 4$) is the channel loss exponent.

  – d_{ij} is the Euclidean distance between nodes i and j.

Problem Statement

- s is the source node.
- $\mathcal{N}$ is the set of all nodes in the network, cardinality N.
- $\mathcal{E}$ is the set of all directed edges, cardinality E.
- $\mathcal{D} \subseteq \{\mathcal{N} \setminus s\}$ is the set of destination nodes, cardinality D.
- Denoting the transmitter power threshold of node i by Y_i^{max}, $\mathcal{E}$ is given by:

$$\mathcal{E} = \{(i \rightarrow j) \mid (i, j) \in \mathcal{N},\ i \neq j,\ \mathbf{P}_{ij} \leq Y_i^{max},\ j \neq s\} \qquad (2)$$

Problem Statement

- $\{F_{ij} : \forall(i \rightarrow j) \in \mathcal{E}\}$ is a set of flow variables.
- $\{H_i : \forall i \in \mathcal{N}\}$ is a set of binary variables denoting *hop-count*.
- For wired networks, the hop-count of any node i, H_i, is the number of links carrying positive flow out of the node.
- For wireless networks H_i is an indicator variable – equal to 1 if there is at least one link carrying a positive flow out of node i, and 0 otherwise.
- This definition is due to the fact that multiple nodes can be reached from a transmitting node using a single transmission to the farthest node.
- Total hop-count is thus the number of transmitting nodes in the multicast tree.
- Minimizing the total hop-count is equivalent to minimizing the number of transmitting nodes in the tree.

Problem Statement

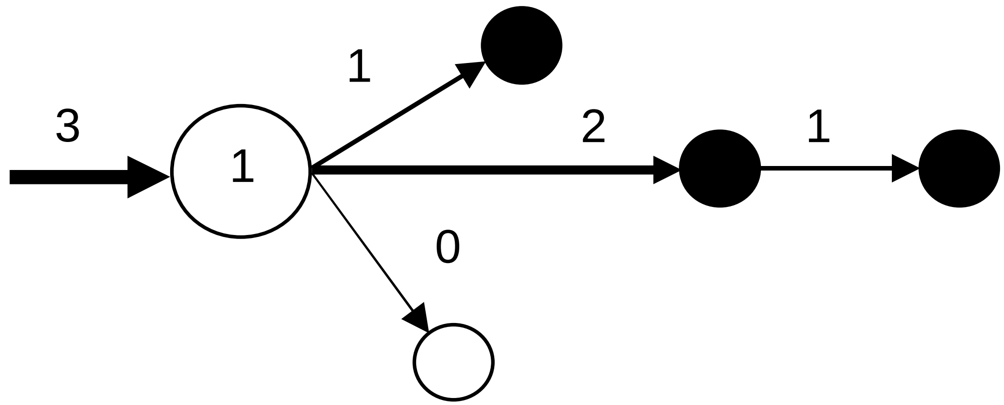

Figure 1: Shaded circles represent the destination nodes. The numbers above the edges are the flows. For a wired network, the hop-count of node 1 is 2, equal to the number of edges directed out of node 1 carrying a positive flow. If the network is wireless and if nodes have omni-directional antennas, the hop-count of node 1 is 1, since it can send a packet to the farther destination node, which will be picked up by the destination node closer to it. Thus the total hop-count in a wireless multicast tree is equal to the number of transmitting nodes in the tree.

Mathematical Model

The objective function of the minimum-hop multicast problem in wireless networks can be written as:

$$\text{minimize} \sum_{i=1}^{N} H_i \tag{3}$$

- This problem can be interpreted as a single-commodity, single-origin multiple-destination uncapacitated flow problem, with the source having D units of supply and the destination nodes having one unit of demand each.
- For other nodes, the net in-flow must equal the net out-flow, since they serve only as relay nodes (note that not all of the relay nodes need to act as such).
- This model can also be viewed as a token allocation scheme where the source node generates as many tokens as there are destination nodes and distributes them along the "most efficient" (in terms of number of hops) tree such that each destination node gets to keep one token each.

Mathematical Model

This problem can be solved using the usual *conservation of flow constraints*:

$$\begin{aligned}
\sum_{j=1}^{N} F_{ij} &= D; \quad i = s, (i \to j) \in \mathcal{E} && (4) \\
\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} &= 1; \quad \forall i \in \mathcal{D}, (i \to j) \in \mathcal{E} && (5) \\
\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} &= 0; \quad \forall i \notin \{\mathcal{D} \cup s\}, (i \to j) \in \mathcal{E} && (6)
\end{aligned}$$

Mathematical Model

Constraints linking the flow variables to the hop-count variables are given by

$$D \cdot H_i - \sum_{j=1}^{N} F_{ij} \geq 0; \; \forall i \in \mathcal{N}, \; (i \to j) \in \mathcal{E} \tag{7}$$

- Eq. (7) says that “the hop-count of a node is equal to 1 if there is a positive flow in at least one link directed away from the node, and 0 otherwise”.
- The coefficient of H_i above comes about since the maximum flow out of a node is equal to the number of destination nodes.

Mathematical Model

- The final set of constraints express the integrality of the H_i variables and non-negativity of the F_{ij} variables.

$$H_i \in \{0, 1\}; \ \ \forall i \in \mathcal{N} \tag{8}$$

$$F_{ij} \geq 0; \ \ \forall (i \rightarrow j) \in \mathcal{E} \tag{9}$$

- In summary, the objective function in Eq. (3) subject constraints in Eqs. (4) to (9) solves the minimum-hop multicast problem in wireless networks.

- There are $E + N$ variables (the number of flow variables in the formulation is equal to E, and the number of hop-count variables is equal to N.

- Strictly speaking, however, the number of hop-count variables is equal to $N - 1$ since the multicast tree must include a transmission from the source and hence H_i must be equal to 1 for $i =$ *source*.

Discussion

- The routing tree is constructed by identifying the transmitting nodes and their farthest neighbors for which there is an outward positive flow.
- Let node 1 be the source and the destination nodes be 2, 5, 7, 9 and 10.

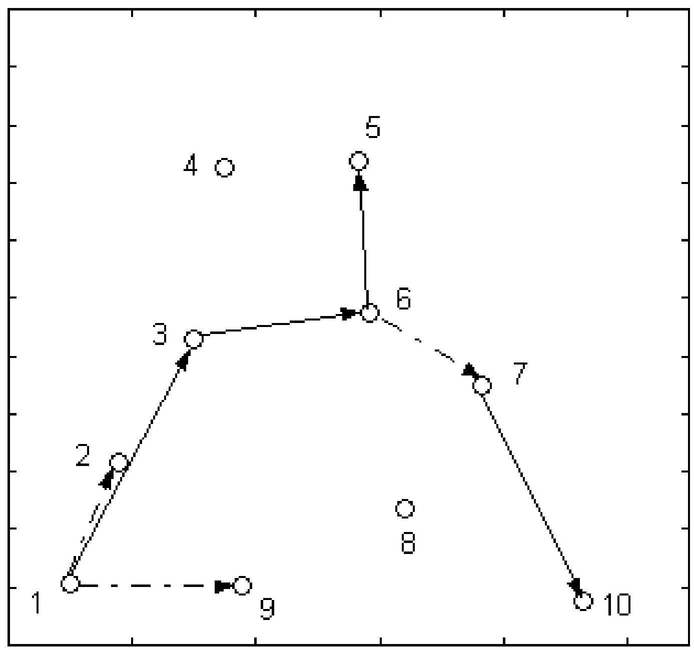

Figure 2: Example 10-node network with node degree of connectivity = 3.

Discussion

- The solid lines represent the actual transmissions in the multicast tree. The dotted lines represent *implicit* transmissions; *i.e.*, the associated recipient nodes pick up the transmissions by virtue of their being closer to the
- The flow variables and their optimal values are shown below:

$$\mathbf{F} = \begin{bmatrix} - & 1 & 3 & - & - & - & - & - & 1 & - \\ - & - & 0 & - & - & - & - & - & 0 & - \\ - & 0 & - & 0 & - & 3 & - & - & - & - \\ - & - & 0 & - & 0 & 0 & - & - & - & - \\ - & - & 0 & 0 & - & 0 & - & - & - & - \\ - & - & 0 & - & 1 & - & 2 & - & - & - \\ - & - & - & - & - & 0 & - & 0 & - & 1 \\ - & - & - & - & - & - & 0 & - & 0 & 0 \\ - & 0 & - & - & - & - & - & 0 & - & - \\ - & - & - & - & - & - & 0 & 0 & 0 & - \end{bmatrix} \tag{10}$$

- The first column in the flow matrix, $\mathbf{F}$, is empty since node 1 is the source and reflects the condition $j \neq$ *source* in Eq. (2).
- Diagonal elements of $\mathbf{F}$ are empty because of the condition $i \neq j$ in Eq. (2).
- Whether flow variables corresponding to the rest of the indices exist or not is dictated by the maximum power constraint on the transmitters.
- Examining the first row of the optimal flow values in Eq. (10), we see that there are non-zero flows from node 1 to nodes 2, 3 and 9, of which node 3 is the farthest.
- This is shown as a solid line from node 1 to 3 in Figure 2.
- The dotted lines to nodes 2 and 9 represent that these nodes pick up the transmission by virtue of their being closer to node 1 than 3.
- The actual sequence of transmissions is therefore: $\{1 \rightarrow 3, 3 \rightarrow 6, 6 \rightarrow 5, 7 \rightarrow 10\}$.

Discussion

- The above MILP model can also be used for obtaining *maximum power constrained* minimum hop multicast trees.
- In [Das. et. al., *Globecom 2003*], a polynomial time optimal algorithm was presented for obtaining multicast trees which maximize minimum node lifetime, or alternately, as a special case, minimize the maximum transmit power.
- Let $\hat{Y}$ be the optimal maximum transmit power obtained after solving the minimax problem. Redefining the set of valid edges as:

$$\mathcal{E} = \{(i \rightarrow j) \mid (i,j) \in \mathcal{N},\ i \neq j,\ \mathbf{P}_{ij} \leq \hat{Y},\ j \neq s\} \tag{11}$$

in place of Eq. (2) and solving the MILP model will yield a minimum hop multicast tree such that the maximum transmit power is not greater than $\hat{Y}$.

LP-relaxation

- Such relaxations usually form the basis of approximation algorithms.
- Given an MILP problem **P** and an instance of the problem, I, we denote the LP-relaxation of the instance by $LP(I)$.
- If the optimal solution of $LP(I)$ is integral, the problem is solved.
- Otherwise, the fractional optimal solution, which is a lower bound on the optimal solution of I, is usually rounded (which can be deterministic or randomized) to provide a feasible integral solution.
- (There exists other methods for converting the fractional solution to an integral solution, $e.g.$, the primal-dual scheme).
- An approximation guarantee can then be obtained by comparing the costs of the fractional solution and the integral solution.

LP-relaxation

- LP-relaxation of our model is obtained by replacing constraints in Eq. (8) with:

$$0 \leq H_i \leq 1; \;\; \forall i \in \mathcal{N} \tag{12}$$

- From Eq. (7), we see that H_i will be set equal to $\sum_j F_{ij}/D$ in the optimal solution of the relaxed model since the objective function involves minimization of the sum of all H_i's.

- Note that $0 \leq \sum_j F_{ij}/D \leq 1$ since the maximum flow out of any node is equal to D and the minimum is 0.

- Therefore, the upper bound on H_i in (12) is redundant and the variables $\{H_i\}$ can simply be declared to be non-negative, as shown below:

$$H_i \geq 0; \;\; \forall i \in \mathcal{N} \tag{13}$$

LP-relaxation

- If there is zero flow out of node i, H_i will be equal to 0 in the optimal solution; *i.e.*, $H_i = 0$ if $\sum_j F_{ij} = 0, j \neq i$.
- No rounding is therefore required in this case for H_i.
- Since net flow out of the source node is always equal to D, H_{source} will be equal to 1 in the optimal relaxed solution and will not require any rounding.
- For any node $i \neq$ *source*, if the total outflow is non-zero and the ratio $\sum_j F_{ij}/D$ is fractional ($i.e.$, $0 < \sum_j F_{ij} < D$), the cost associated with rounding up H_i to the nearest integral value is $1 - \sum_j F_{ij}/D$.
- The round-up cost associated with the node H_i decreases as $\sum_j F_{ij} \rightarrow D$.
- Maximum round-up cost occurs when $\sum_j F_{ij} = 1$.

LP-relaxation

We now construct a problem instance for which the optimal relaxed solution will incur the maximum round-up cost. The round-up cost will be maximum if the following conditions are satisfied:

(a) The optimal minimum-hop multicast tree comprises of node-disjoint (except at the source node) paths to each of the destination nodes,

(b) All nodes other than source and destinations are used as relays and carry unit flow each This must be satisfied since the paths to the destinations are node-disjoint, except at the source.

(c) All destination nodes are leaves in the optimal tree and are farthest from the source (in terms of number of hops). For odd N ($N \geq 5$), this condition can be met if the number of destination nodes is given by $D = (N-1)^{1/2}$.

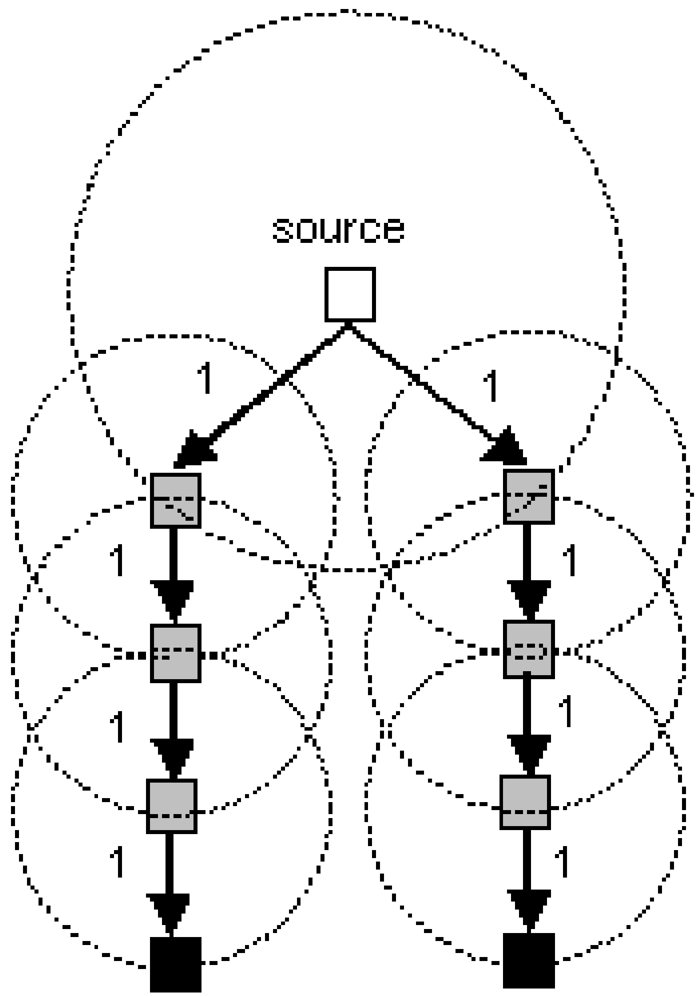

Figure 3: A 9-node, 2-destination problem for which the round-up cost incurred in converting the optimal fractional solution to an integral solution is the maximum.

LP-relaxation

- The darkly shaded nodes are the destinations and the dotted circles represent the communication range of each node.
- The optimal solution for this problem instance involves 7 hops, as shown.
- If the LP-relaxation of this problem is solved, all directed links will be assigned unit flow as shown.
- Correspondingly, the optimal cost of the relaxation is equal to $1 + 6 \times (1/2) = 4$, since the hop-count of the source is equal to $\sum_j F_{ij}/D = 2/2 = 1$ and that of all relay nodes (shown lightly shaded) is equal to $\sum_j F_{ij}/D = 1/2$. The round up cost is 7 (7 transmit nodes), which is equal to the number of hops.
- If the above conditions are satisfied, it can be shown that the ratio of the optimal solution to its LP-relaxation is given by $D(N-D)/(N-1) \leq D$.

Sequential Shortest Path heuristic

- A sub-optimal sequential shortest path heuristic for solving the MILP problem.
- Let π_D be any ordering of the destination nodes with respect to the source.
- For example, they can be ordered with respect to increasing or decreasing Euclidean distance from the source, or order the destination nodes on the basis of a shortest path (in terms of number of hops) tree to the source.
- The MILP problem is solved by computing a series of shortest paths in the sequence given by π_D.

Sequential Shortest Path Heuristic

- Let $\mathbf{W}^{(1)}$ be the initial weight matrix used for computing the shortest path between the source and $\pi_D(1)$, the first node in π_D.
- The $(i, j)th$ element of $\mathbf{W}^{(1)}$ is given by:

$$\mathbf{W}_{ij}^{(1)} = \begin{cases} 1, & \text{if } (i \longrightarrow j) \in \mathcal{E} \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

where $\mathcal{E}$ is the initial set of edges defined in Eq. (2).

Sequential Shortest Path Heuristic

- The minimum hop multicast problem in wireless networks with omni-directional antennas can be viewed as a minimization of the number of transmitting nodes.
- Consequently, if $\vec{tr}_1$ is the set of transmitting nodes in the shortest path obtained after the first iteration, using these nodes as relays in subsequent iterations would not incur any additional cost.
- Thus, the weight matrix for the second iteration can be modified as follows:

$$\mathbf{W}_{ij}^{(2)} = \begin{cases} 0, & \text{if } i \in \vec{tr}_1 \\ \mathbf{W}_{ij}^{(1)}, & \text{otherwise} \end{cases} \tag{15}$$

Sequential Shortest Path Heuristic

- We refer to the weight modification procedure after each iteration as *node unwrapping*.
- Using $\mathbf{W}^{(2)}$, a shortest path is computed between the source and $\pi_D(2)$, the second node in π_D.
- This procedure is repeated till all destination nodes are reached and the final multicast tree is obtained by concatenating the shortest paths obtained at each iteration.

Sequential Shortest Path Heuristic

1. Let π_D be any ordering of the destination nodes with respect to the source.
2. Let k be the iteration index.
3. Let $path(k)$ be the shortest path obtained at iteration k.
4. Set $k = 1$;
5. Compute the initial weight matrix $\mathbf{W}^{(k)}$ (see eqn. 14).
6. Find the shortest path between the $source$ and the node $\pi_D(k)$, $path(k)$.
7. $\mathbf{while}$(not all destination nodes reached)
   - Increment $k = k + 1$;
   - Compute the new weight matrix $\mathbf{W}^{(k)}$ (see eqn. 15).
   - Find the shortest path between the $source$ and $\pi_D(k)$.

   $\mathbf{end\ while}$
8. Concatenate the set of shortest paths $\{path(k)\}$ to obtain the multicast tree.

Figure 4: High level description of the sequential shortest path algorithm.

Sequential Shortest Path Heuristic

- Since distributed algorithms (such as distributed Bellman-Ford) exist for the shortest path problem, the heuristic is amenable to distributed implementation provided the multicast group members are aware of their Euclidean distance (or, any other criterion used to sort the destination nodes) from the source.
- The algorithm can also be used for maximum power constrained minimum hop multicasting if the set of directed edges in the underlying graph is defined as in Eq. (11).
- The above procedure would take D shortest path iterations to terminate, one iteration for every destination.

Sequential Shortest Path Heuristic

- However, because of node unwrapping, it may be possible to reach additional destination nodes without any additional cost, as illustrated below.

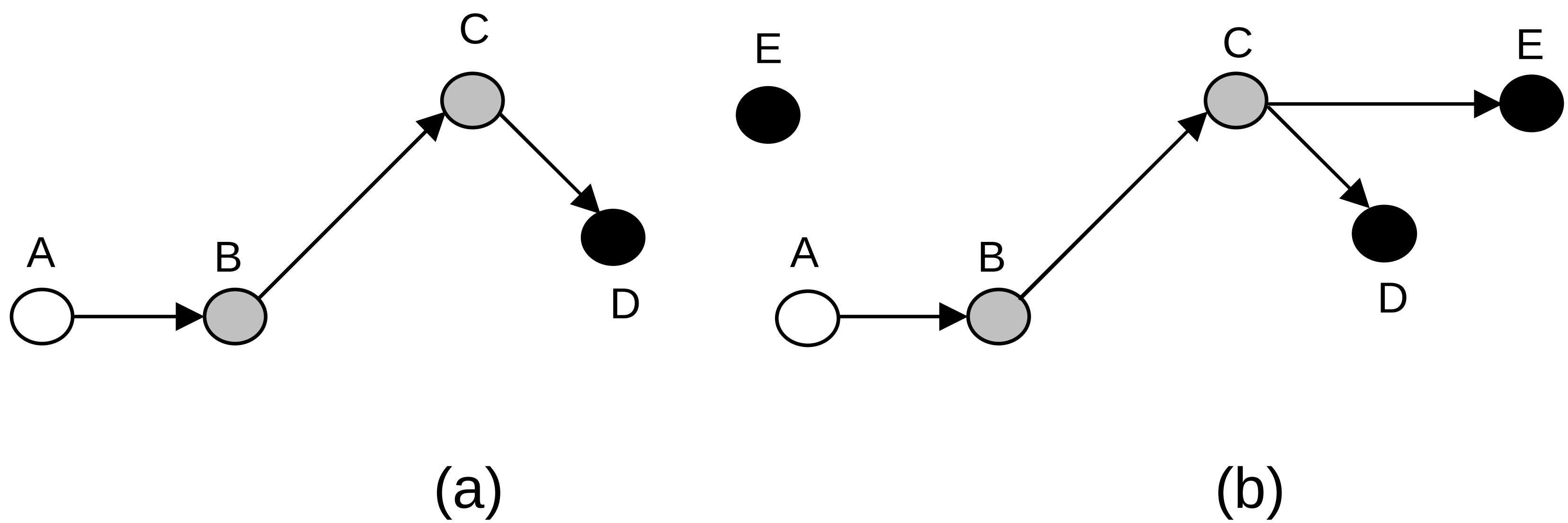

Figure 5: (a) Shortest path at current iteration, $A \longrightarrow B \longrightarrow C \longrightarrow D$, before node-unwrapping. (b) Node E can be reached simply by unwrapping node C. No additional iteration is required.

Sequential Shortest Path Heuristic

- A simple modification to the algorithm in Figure 4 can be made to check whether additional destination nodes can be reached by node unwrapping.
- If so, those destination nodes that have not yet been reached after unwrapping can be reordered and the first node in the reordered set chosen as the destination for the next shortest path iteration.
- Experimental results suggest that ordering the destination nodes with respect to decreasing Euclidean distance from the source (*i.e.*, the farthest node is the destination for the first iteration) usually results in the fewest number of iterations than if they are ordered with respect to increasing Euclidean distance, with no appreciable difference in solution quality.
- As implemented, if there are multiple shortest paths at any iteration with the same hop count, any one is chosen arbitrarily.

Simulation Results

- Studied different multicast group sizes in 20, 30, 40 and 50-node networks.
- Networks and destination sets were chosen so that a feasible solution exists (*i.e.*, a solution where all destination nodes can be reached, given the transmitter power constraints.)
- Transmitter power constraints were set so that each node was connected to its 4 nearest neighbors.
- LPSOLVE which uses a branch and brand algorithm to solve MILP problems, was used to compute the optimal solutions.

Simulation Results

- The sequential shortest path algorithm was implemented by ordering the destination nodes with respect to decreasing Euclidean distance from the source.

- The performance measures for comparing the optimal and heuristic solutions are the *mean* (PM_1), *max* (PM_2) and *standard deviation* (PM_3) of the ratio of the sequential shortest path heuristic to the optimal, over 50 randomly generated instances.

- Table 1 provides a statistical summary of the simulation results for multicast group sizes 5, 10 and 15.

- The heuristic performs quite reasonably on average, being within 110% of optimal in all cases. Worst performance is for 20-node networks and multicast group size = 5, where the heuristic hop count is 140% of the optimal hop count.

Table 1: Simulation results.

N	*Multicast Size*	PM_1	PM_2	PM_3
20	5	1.06	1.40	0.12
	10	1.05	1.25	0.08
	15	1.09	1.30	0.10
30	5	1.04	1.38	0.09
	10	1.05	1.20	0.06
	15	1.05	1.22	0.06
40	5	1.04	1.25	0.07
	10	1.04	1.20	0.06
	15	1.07	1.20	0.06
50	5	1.03	1.22	0.06
	10	1.06	1.27	0.08
	15	1.09	1.31	0.08

Conclusion

- We have presented a mixed integer linear programming model and a sub-optimal sequential shortest path heuristic for solving the minimum-hop multicast problem in wireless networks with omni-directional antennas.
- We have also showed that a simple redefinition of the set of directed edges in the network graph allows for the solution of the minimum hop multicast problem subject to a maximum transmitter power constraint.
- The heuristic algorithm has been shown to perform reasonably well in simulations conducted on different multicast group sizes in small and medium scale networks.
- We are currently working on incorporating QoS (bounded delay and minimum SINR) guarantees in the MILP model.